

Overview of My Research Activities in Applied Mathematics

Presented by Dr. Ronan Dupont
July 1, 2025

Interview for the Position:

Postdoctoral Position in Numerical Linear Algebra *Nagoya University, Japan*

Moonshot R&D Program – “Backcasting digital system by super-dimensional state engineering”

Supervised by Dr. Shao-Liang Zhang and Dr. Tomohiro Sogabe

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- 1 Presentation of Myself
- 2 Pre-thesis Research Projects
- 3 Thesis Research Projects
- 4 Post-Thesis Research Projects

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1 Presentation of Myself

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- ▷ Academic Origins
- ▷ PhD
- ▷ Post-PhD Phase
- ▷ Beyond Mathematics
- ▷ Any questions?

2 Pre-thesis Research Projects

3 Thesis Research Projects

4 Post-Thesis Research Projects

It starts with 3 years of living in Japan...

- Born to a nuclear engineer (father) and a Mathematics teacher (mother).

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- Born to a nuclear engineer (father) and a Mathematics teacher (mother).
- First expatriation: Rokkasho nuclear power plant project, Japan.



Figure: Me at Japanese School.

Academic Foundations



Highly selective classes to prepare for the competitive exams (2016–2018)

- Intensive mathematics and physics curriculum.
- Solid foundations in algebra, analysis, mechanics, and modeling.

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- Dual track: **CFD and Applied Mathematics + Marine Science.**
- Developed strong interest in numerical methods (finite element, finite volume, ...).

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- Lifeguard on beaches in Normandy (8 Summers).
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After these graduations, the desire to continue abroad was there, but it was the COVID year.

PhD in Montpellier – Coupling Waves and Morphodynamics

A natural continuation of my desire to stay at the interface between mathematics and marine sciences.



PhD in Applied Mathematics – Montpellier (2021–2024)

- Wave-morphodynamic coupling of the coastline by minimization principle.
- Published results in peer-reviewed journals; presented at international conferences.
- Supervised by Pr. Bijan Mohammadi and Pr. Frédéric Bouchette (Interface of Mathematics and Geosciences).
- Teaching experience at the University of Montpellier in : Algebra, Calculus, Cardinality, Geometry, Python, Coastal Modeling.

I felt the need to work on more mathematical subjects.

Post-PhD Path: Teaching and Mobility

Teaching in Bordeaux (2025)

- High-school mathematics teacher in France.
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Move to Asia – Cambodia (2025–)

- Currently teaching at the French International School of Phnom Penh.
- Classes for both **French** (Brevet, Baccalauréat) and **British** (A-level Further Maths) curricula.



My first expatriation was a rewarding success, but I now seek renewed intellectual challenges in Asia.

Passions and Personal Interests

Personal Interests

- **Surfing, swimming, snorkeling** — strong personal connection to the sea; spent 8 summers as a beach lifeguard.
- **Travel** — lived in Japan as a child; currently in Cambodia; completed a solo tour across Europe after my PhD.
- **Languages** — fluent in French and English and almost Italian; currently studying Khmer since my arrival in Phnom Penh.
- **Scientific curiosity** — I enjoy learning new algorithms, coding daily, and sharing math-code challenges with my students.

This path naturally led me to apply for a position in Japan — where my scientific story began.

Any questions?



More details:



My Website and my CV:

ronan-dupont.github.io

ronan-dupont.github.io/files/Curriculum_Vitae_Ronan_Dupont.pdf

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1 Presentation of Myself

2 Pre-thesis Research Projects

- ▷ Numerical Linear Algebra
- ▷ Modeling and Numerical Simulation of Epidemics and Diffusion
- ▷ Stochastic Computation
- ▷ Mesh Generation and Visualization
- ▷ Any questions?

3 Thesis Research Projects

4 Post-Thesis Research Projects

Linear Solvers, Iterative Methods and Equation Solving

Direct and Iterative Methods for Linear Systems

- LU factorization implemented from scratch.
- Iterative solvers: Jacobi and Gauss-Seidel methods.
- Gradient and Conjugate Gradient methods.

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Numerical Schemes for AD Equations (1D/2D)

- Explicit/implicit schemes for AD equations and FreeFem++.
- Resolution using a deterministic approach (finite difference resolution, finite element, finite volume) and a stochastic approach (Monte Carlo, quasi-Monte Carlo, Brownian motion).

Modeling Epidemics

SIR and SZR Models

- Resolution of PDEs with diffusion using finite differences.
- Classical SIR model, playful extension to SZR (Zombie outbreaks).
- Visual simulations with spatial propagation and mapping.

Jour= 450

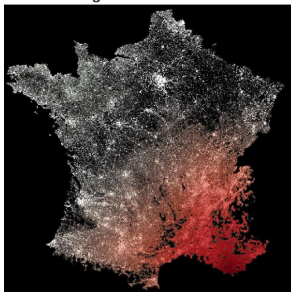


Figure: Day 450 of a Zombie epidemic starting from Toulon (south of France).

Optimization, Stochastic Methods and Quantization Algorithms

CFD optimization of the performance of windsurf sails intended for high speeds

- 6-month internship performing CFD / FSI calculations.
- Development of new windsurf sail types.

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- Vector quantization with K-means and Kohonen algorithms.

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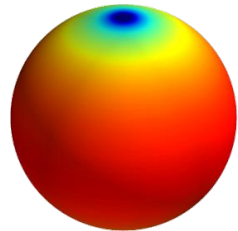
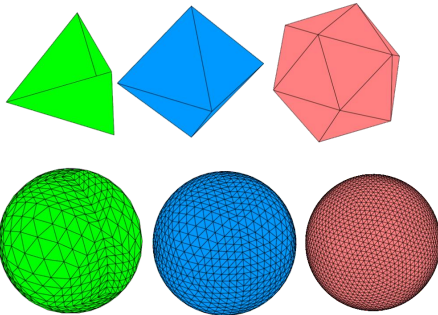
Stochastic Problem Solving and Optimization

- Solving Sudoku puzzles using genetic algorithms.
- Solving function minima by genetic algorithm.
- Integral computation via Monte Carlo / Quasi Monte Carlo integration.
- Stochastic resolution of PDEs.

Development of a Meshing Tool for Spheres

Mesh Generation in Fortran

- Surface meshing of a sphere (2D) in Fortran 90.
- Custom types and coordinate storage.
- Visualization in Python.
- Application to thermal diffusion equations.



Any questions?



More details:



Master projects reports:

`ronan-dupont.github.io/publication/2021-master-reports`

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- ▷ Modelling beaches morphodynamic by Hadamard sensitivity analysis
- ▷ How does the model (OptiMorph) work?
- ▷ How to compute $\nabla_{\psi} \mathcal{J}$?
- ▷ Some extensions of the model
- ▷ Any questions?

4 Post-Thesis Research Projects

Modelling beaches morphodynamic by Hadamard sensitivity analysis

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Publications:



Ronan Dupont, Frédéric Bouchette, and Bijan Mohammadi. (2024). "Beaches morphodynamic modeling based on Hadamard sensitivity analysis." In: *Ocean Modelling*, doi.org/10.1016/j.ocemod.2024.102370.



Ronan Dupont, Megan Cook, Frédéric Bouchette, Bijan Mohammadi, and Samuel Meulé. (2023). "Sandy beach dynamics by constrained wave energy minimization." In: *Ocean Modelling*, doi.org/10.1016/j.ocemod.2023.102197.

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Main model assumption

Nature adapts the seabed ψ so that wave energy is minimized E_H , ie:

$$\min_{\psi} \mathcal{J}(\psi) \quad \text{with} \quad \mathcal{J}(\psi) = \int_{\Omega} E_H(\psi),$$

Workflow based on the Hadamard shape derivative

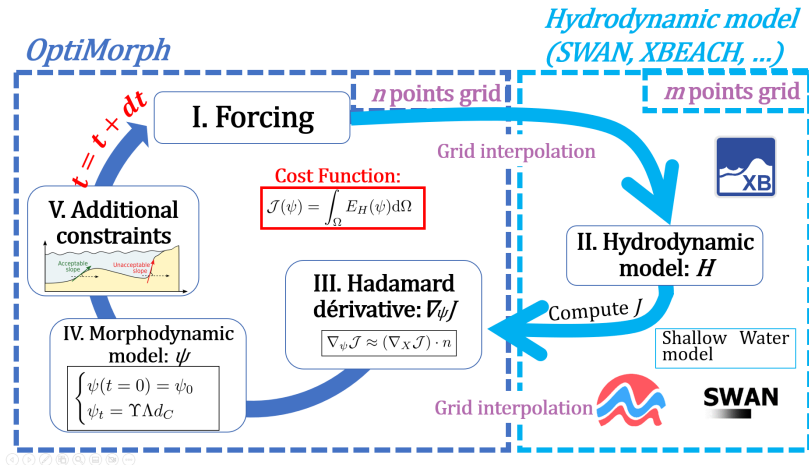


Figure: OptiMorph workflow based on Hadamard shape derivative.

Morphodynamic model

Problem reminder:

We aim to solve $\min_{\psi} \mathcal{J}(\psi)$ under physical constraints.

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Without physical constraints:

$$\begin{cases} \psi_t = \Upsilon d \\ \psi(t = 0) = \psi_0 \end{cases}$$

where:

- $\psi(t)$: seabed profile at time t ,
- Υ : sediment mobility,
- d : descent direction. Here, $d = -\nabla_{\psi} \mathcal{J}$.

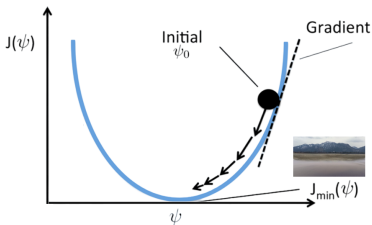


Figure: Gradient descent at $t = 0$ s.

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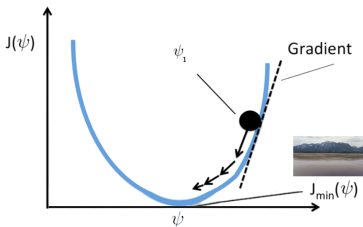


Figure: Gradient descent at $t = \Delta t$ s.

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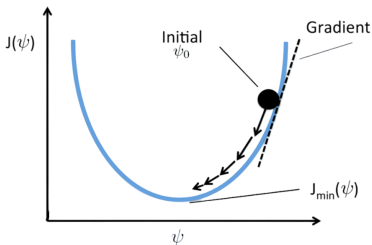


Figure: Gradient descent with constraints.

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- $\psi(t)$: seabed profile at time t ,
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- $\wedge(x)$: wave-induced excitation of the seabed.

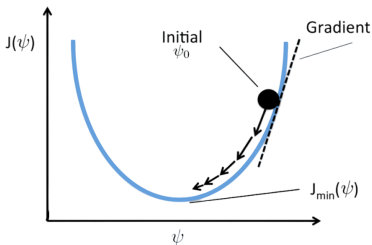


Figure: Gradient descent.

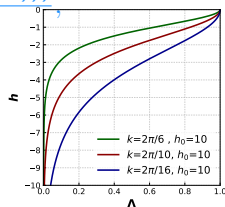
Adding physical constraints

- Sand excitation. From (Soulsby, 1987):

$$\begin{aligned} \varphi : \Omega \times [0, h_0] &\longrightarrow \mathbb{R}^+ \\ (x, z) &\longmapsto \frac{\cosh(k(x)(h(x) - (h_0 - z)))}{\cosh(k(x)h(x))} \end{aligned}$$

and at $z = \psi$:

$$\Lambda(x) = \varphi(x, \psi(x)) = \frac{1}{\cosh(k(x)h(x))}$$



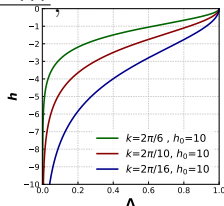
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$$\left| \frac{\partial \psi}{\partial x} \right| \leq M_{\text{slope}}$$

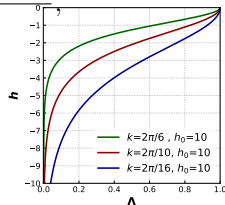
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- Sandstock conservation (in closed domain only):

$$\int_{\Omega} \psi(t, x) dx = \int_{\Omega} \psi_0(x) dx \quad \forall t \in [0, T_f]$$

Projection of the sediment conservation constraint

We aim to compute the descent direction d_C without violating the sediment conservation constraint. We define a residue $C_{\text{sand}}(t)$ which we want to be zero $\forall t \in [0, T_f]$.

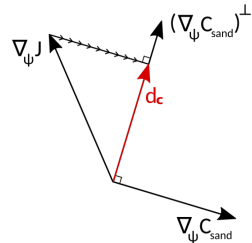


Figure: Projection. Cook (2021).

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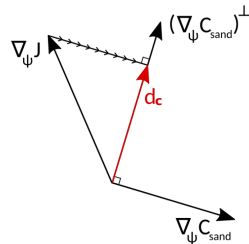


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So we have:

$$d_C = \nabla_{\psi} \mathcal{J} - \left\langle \nabla_{\psi} \mathcal{J}, \frac{\nabla_{\psi} C_{\text{sand}}}{\|\nabla_{\psi} C_{\text{sand}}\|} \right\rangle \frac{\nabla_{\psi} C_{\text{sand}}}{\|\nabla_{\psi} C_{\text{sand}}\|}.$$

How to compute $\nabla_{\psi}\mathcal{J}$? Different strategies

Reminder of the governing equation:

$$\begin{cases} \psi_t = \Upsilon \wedge d_C \\ \psi(t=0) = \psi_0 \end{cases} \quad \begin{cases} d_C = -\nabla_{\psi}\mathcal{J} + \text{constraints}, \\ \mathcal{J}(\psi) = \int_{\Omega} E_H(\psi) \end{cases}$$

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- ✓ Exact solution.
- ✓ Fast.

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- ✗ Requires $N + 1$ evaluations (each time step).
- ✗ Very long computation time.

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Automatic Differentiation



- ✓ Robust.

✗ Heavy pre-processing.
✗ Depends on C / Fortran 90.

Mohammadi & Bouchette (2014)

Hadamard derivative of $\nabla_{\psi}\mathcal{J}$

We consider:

$$\nabla_{\psi}\mathcal{J} = \lim_{\varepsilon \rightarrow 0} \frac{\mathcal{J}(\psi + \varepsilon n) - \mathcal{J}(\psi)}{\varepsilon}$$

with n : normal vector to the shape.

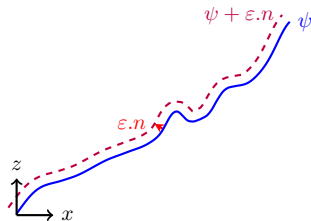


Figure: Illustration of Hadamard derivative.

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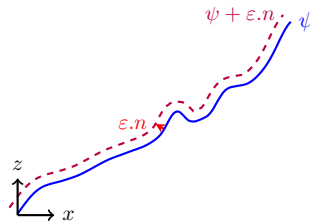


Figure: Illustration of Hadamard derivative.

At first order:

$$\nabla_{\psi} \mathcal{J} \approx \lim_{\varepsilon \rightarrow 0} \frac{\cancel{\mathcal{J}(\psi)} + \varepsilon \nabla_X \mathcal{J} \cdot n - \cancel{\mathcal{J}(\psi)}}{\varepsilon}, \quad \text{with } X = (x, z)^T.$$

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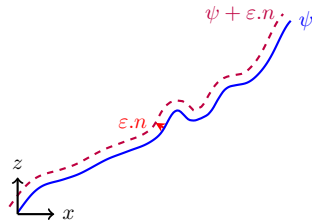


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This gives:

$$\boxed{\nabla_{\psi} \mathcal{J} \approx (\nabla_X \mathcal{J}) \cdot n}$$

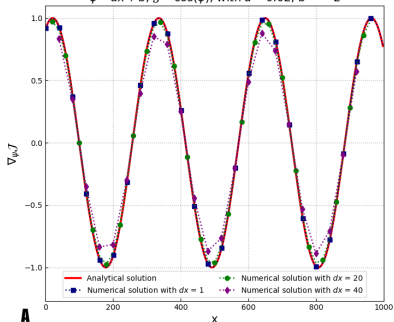
Numerical validation on an analytical case

We consider:

$$\psi = \{(x, y) \in \mathbb{R}^2 \mid y = ax + b\}$$

and $\mathcal{J} = \cos(\psi)$, with $\nabla_{\psi} \mathcal{J} = -\sin(\psi) \sqrt{a^2 + 1}$.

$\nabla_{\psi} \mathcal{J}$ using Hadamard approximation with the following problem:
 $\psi = ax + b$, $\mathcal{J} = \cos(\psi)$, with $a = 0.02$, $b = -2$



Error ε_{L^2} between the analytical and numerical solution of the Hadamard approximation - Logarithmic scale

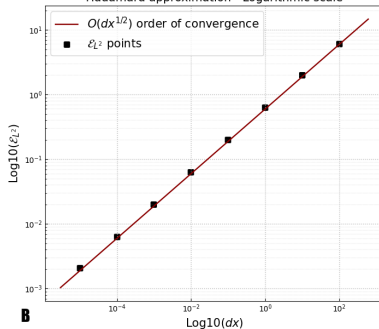


Figure: A) Analytical and approximate solution using the derivative...

Numerical verification on an application case with perturbations

With $\nabla_{\psi} \mathcal{J} \approx \nabla_X \mathcal{J} \cdot n$

Parameters:

- Shoaling model with:
 - $H_0 = 2$ m,
 - $T_0 = 10$ s,
 - $h_0 = 10$ m.
- Linear bottom with perturbations.

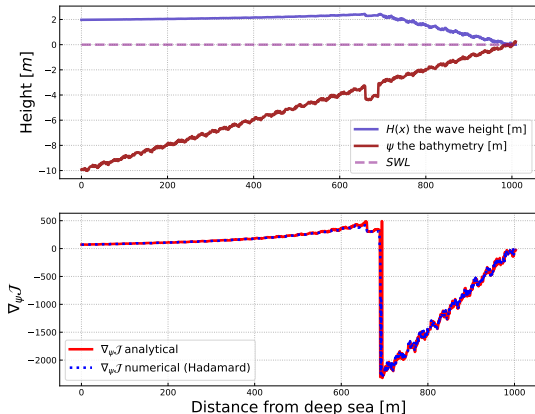


Figure: Computation of $\nabla_{\psi} \mathcal{J}$ using the Hadamard approach (blue) and analytical calculation (red). Linear bottom with perturbations.

Adding lateral transport

- New governing equation: $\psi_t + V \nabla_s \psi = \Upsilon \Lambda d_C$

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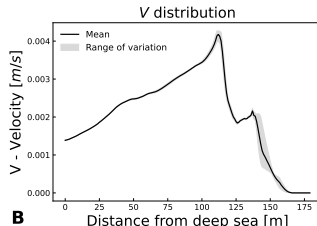
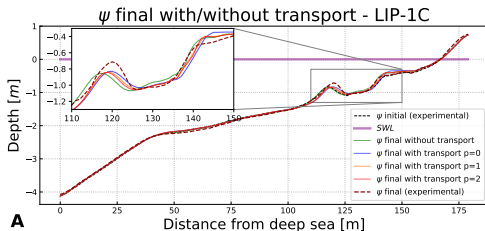


Figure: **A)** OptiMorph morphodynamic results with lateral transport for $p = 0, 1, 2$, and XBeach wave model, for the LIP 1C channel experiment. **B)** Velocity distribution for $p = 1$.

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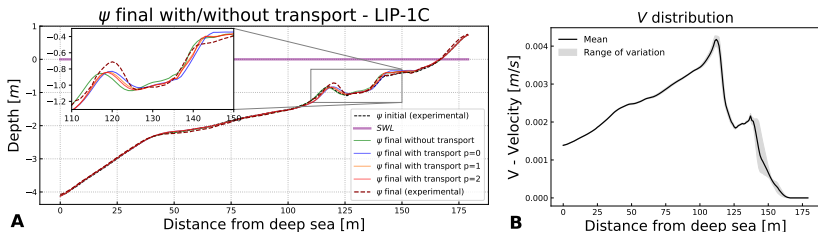


Figure: **A)** OptiMorph morphodynamic results with lateral transport for $p = 0, 1, 2$, and XBeach wave model, for the LIP 1C channel experiment. **B)** Velocity distribution for $p = 1$.

- But $V \geq 0$ for this choice $\Rightarrow$ no offshore sediment bar displacement (as in LIP-1B).

Towards a better definition of velocity?

- A new system of governing equations?

$$\begin{cases} \psi_t + \mathbf{V} \nabla_s \psi = \Upsilon \Lambda d_C & (a) \\ V_t = -\rho \nabla_V \mathcal{J} & (b) \end{cases}$$

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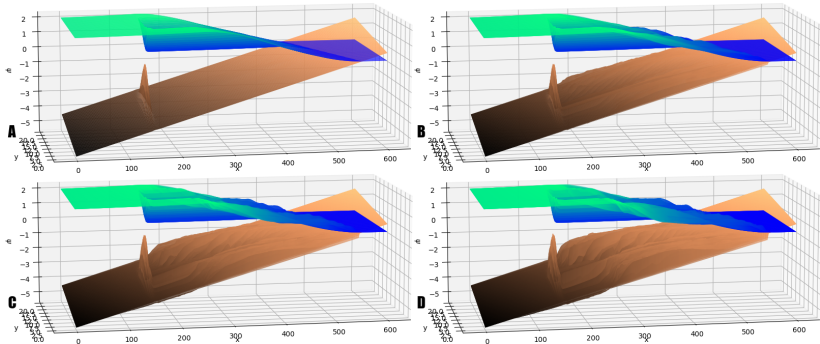
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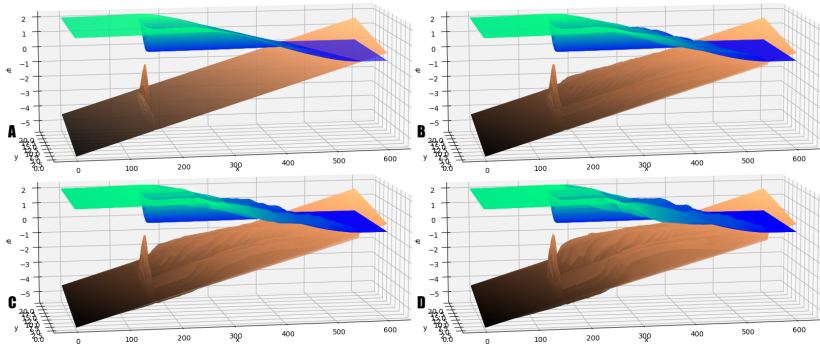
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- Can lateral morphodynamic transport minimize $\mathcal{J}$?

2D Morphodynamic results

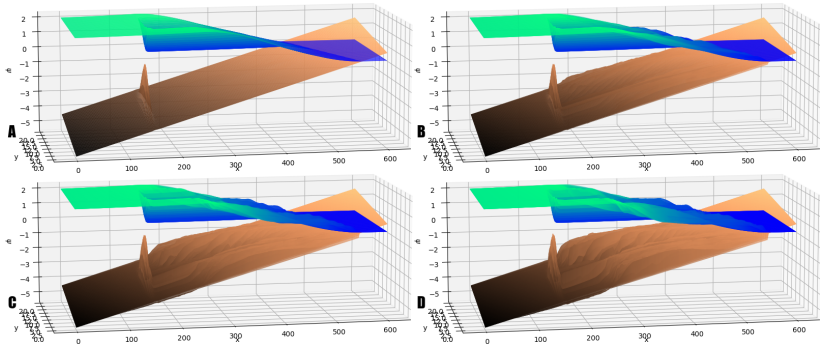


2D Morphodynamic results



- Observation of a sewage pit behind the geotextile tube.

2D Morphodynamic results



- Observation of a sewage pit behind the geotextile tube.
- Seabed modification $\Rightarrow$ appropriate morphodynamic response.

Any questions?



More details:



PhD manuscript:

`ronan-dupont.github.io/publication/2024_manuscript`



PhD defense (oral support):

`ronan-dupont.github.io/talks/phd_defense`

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- ▷ Problem configuration
- ▷ Analytical solution
- ▷ Validation and Convergence
- ▷ Application: Slope sensitivity, Helmholtz vs Mild-Slope
- ▷ Any questions?

An arbitrary-order Virtual Element Method for the Helmholtz equation applied to wave field calculation in port

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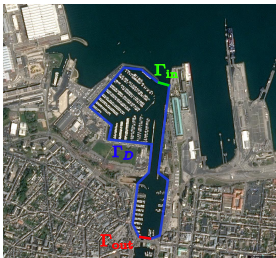
Publication:



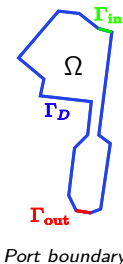
Ronan Dupont. (2025). "An Arbitrary-Order Virtual Element Method for the Helmholtz Equation Applied to Wave Field Calculation in Port." *Results in Applied Mathematics*, doi.org/10.1016/j.rinam.2025.100598.

Problem configuration

A reference to my home town:
Cherbourg in France.



Port location



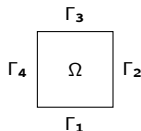
Port boundary

The Helmholtz or Mild-Slope equation:

$$\left\{ \begin{array}{ll} \nabla(C_p C_g \nabla a) + k^2 C_p C_g a = 0, & \text{in } \Omega, \\ \frac{\partial a}{\partial n} = 0, & \text{in } \Gamma_{in} \\ \frac{\partial a}{\partial n} + ik a = 0, & \text{in } \Gamma_{out} \\ a = \gamma a_i & \text{in } \Gamma_D \end{array} \right.$$

Analytical solution

We consider,



$$\begin{cases} \Delta u + k^2 u = f(x, y) & , \text{ in } \Omega, \\ u = u_{\text{exact}} & , \text{ on } \Gamma_2 \cup \Gamma_3 \cup \Gamma_4 \\ \frac{\partial u}{\partial n} + i k u = g(x, y) & , \text{ on } \Gamma_1, \end{cases}$$

with:

$$\begin{cases} u_{\text{exact}}(x, y) = (x + y) \cdot (1 + i) + \exp(x^2 + i y^2), \\ f(x, y) = -((2x)^2 + (2i y)^2 + 2(1 + i)) \cdot \exp(x^2 + i y^2) + k^2 \cdot u_{\text{exact}}(x, y), \\ g(x, y) = (1 + i) + (2i y) \cdot \exp(x^2 + i y^2) + i k \cdot u_{\text{exact}}(x, y). \end{cases}$$

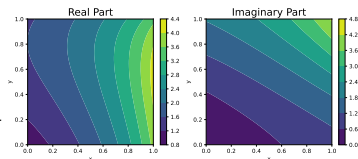


Figure: Real and Imaginary part of u_{exact} .

Validation and Convergence

- Manufactured solution method: exact solution + right-hand side f .
- Implementation tested with $k = 1, 2, 3, 4, 5$ using quadrilateral and polygonal meshes.
- Observed convergence rate:
 - $\|u - u_h\|_{L^2} \sim \mathcal{O}(h^{k+1})$

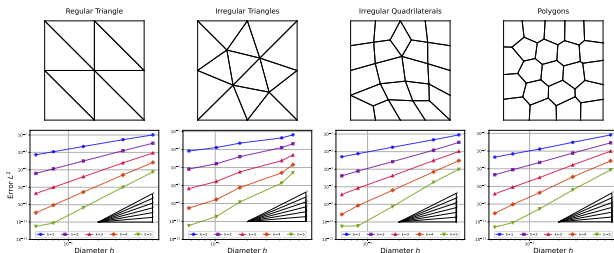


Figure: Convergence curves with different orders k and different types of elements.

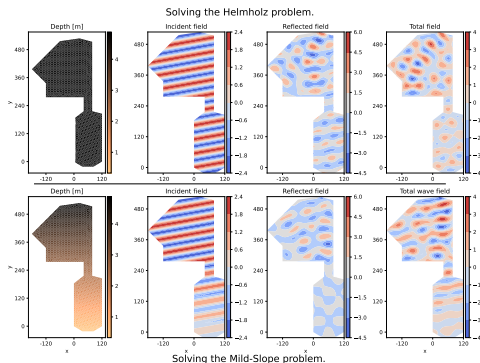
Slope sensitivity, Helmholtz vs Mild-Slope

Problem conditions:

- $a_{\max} = 2 \text{ m}$,
- $T_0 = 8 \text{ s}$,
- $\theta = 280^\circ$.

Points of interest:

- Eigenmode position.



Any questions?



More details:



Ronan Dupont. (2025). "An Arbitrary-Order Virtual Element Method for the Helmholtz Equation Applied to Wave Field Calculation in Port." *Results in Applied Mathematics*, doi.org/10.1016/j.rinam.2025.100598.



Presentation at the NuMerics2024 conference where I was invited (oral support):
ronan-dupont.github.io/talks/NuMerics2024