Overview of My Research Activities in Applied Mathematics

Presented by Dr. Ronan Dupont
July 1, 2025

Interview for the Position:

Postdoctoral Position in Numerical Linear Algebra
Nagoya University, Japan

Moonshot R&D Program – "Backcasting digital system by super-dimensional state engineering"

Supervised by Dr. Shao-Liang Zhang and Dr. Tomohiro Sogabe

Table des Matières

Table of Contents

It starts with 3 years of living in Japan...

- Born to a nuclear engineer (father) and a Mathematics teacher (mother).
- First expatriation: Rokkasho nuclear power plant project, Japan.

Figure: Me at Japanese School.

Academic Foundations

Highly selective classes to prepare for the competitive exams (2016–2018)

- Intensive mathematics and physics curriculum.
- Solid foundations in algebra, analysis, mechanics, and modeling.

SeaTech Engineering School + Marine Sciences MSc (2018–2021)

- Dual track: **CFD and Applied Mathematics** + **Marine Science**.
- Developed strong interest in numerical methods (finite element, finite volume, ...).

Connection with the Sea

- Lifeguard on beaches in Normandy (8 Summers).
- A passion for water sports.

After these graduations, the desire to continue abroad was there, but it was the COVID year.

PhD in Montpellier – Coupling Waves and Morphodynamics

A natural continuation of my desire to stay at the interface between mathematics and marine sciences.

PhD in Applied Mathematics – Montpellier (2021–2024)

- Wave-morphodynamic coupling of the coastline by minimization principle.
- Published results in peer-reviewed journals; presented at international conferences.
- Supervised by Pr. Bijan Mohammadi and Pr. Frédéric Bouchette (Interface of Mathematics and Geosciences).
- Teaching experience at the University of Montpellier in : Algebra, Calculus, Cardinality, Geometry, Python, Coastal Modeling.

I felt the need to work on more mathematical subjects.

Post-PhD Path: Teaching and Mobility

Teaching in Bordeaux (2025)

- High-school mathematics teacher in France.
- Opportunity to try teaching a different audience after teaching at university.

Move to Asia – Cambodia (2025–)

- Currently teaching at the French International School of Phnom Penh.
- Classes for both **French** (Brevet, Baccalauréat) and **British** (A-level Further Maths) curricula.

My first expatriation was a rewarding success, but I now seek renewed intellectual challenges in Asia.

Passions and Personal Interests

Personal Interests

- **Surfing, swimming, snorkeling** — strong personal connection to the sea; spent 8 summers as a beach lifeguard.
- **Travel** — lived in Japan as a child; currently in Cambodia; completed a solo tour across Europe after my PhD.
- **Languages** — fluent in French and English and almost Italian; currently studying Khmer since my arrival in Phnom Penh.
- **Scientific curiosity** — I enjoy learning new algorithms, coding daily, and sharing math–code challenges with my students.

This path naturally led me to apply for a position in Japan — where my scientific story began.

Any questions?

More details:

My Website and my CV:
ronan-dupont.github.io
ronan-dupont.github.io/files/Curriculum_Vitae_Ronan_Dupont.pdf

Table of Contents

Linear Solvers, Iterative Methods and Equation Solving

Direct and Iterative Methods for Linear Systems

- LU factorization implemented from scratch.
- Iterative solvers: Jacobi and Gauss-Seidel methods.
- Gradient and Conjugate Gradient methods.

Nonlinear Systems: Newton-Raphson Method

- Resolution of nonlinear equations using Newton-Raphson.
- Application to nonlinear thermal diffusion problems.

Numerical Schemes for AD Equations (1D/2D)

- Explicit/implicit schemes for AD equations and FreeFem++.
- Resolution using a deterministic approach (finite difference resolution, finite element, finite volume) and a stochastic approach (Monte Carlo, quasi-Monte Carlo, Brownian motion).

Modeling Epidemics

SIR and SZR Models

- Resolution of PDEs with diffusion using finite differences.
- Classical SIR model, playful extension to SZR (Zombie outbreaks).
- Visual simulations with spatial propagation and mapping.

Figure: Day 450 of a Zombie epidemic starting from Toulon (south of France).

Optimization, Stochastic Methods and Quantization Algorithms

CFD optimization of the performance of windsurf sails intended for high speeds

- 6-month internship performing CFD / FSI calculations.
- Development of new windsurf sail types.

Monte Carlo Sampling and Clustering

- Monte Carlo sampling in random fields.
- Vector quantization with K-means and Kohonen algorithms.

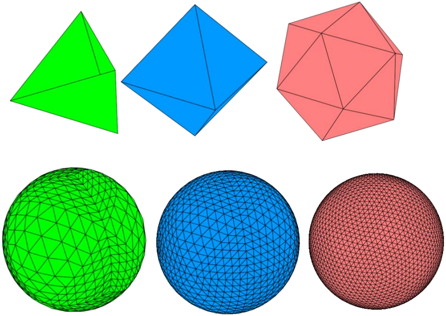
Stochastic Problem Solving and Optimization

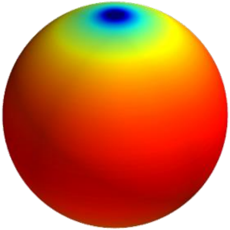
- Solving Sudoku puzzles using genetic algorithms.
- Solving function minima by genetic algorithm.
- Integral computation via Monte Carlo / Quasi Monte Carlo integration.
- Stochastic resolution of PDEs.

Development of a Meshing Tool for Spheres

Mesh Generation in Fortran

- Surface meshing of a sphere (2D) in Fortran 90.
- Custom types and coordinate storage.
- Visualization in Python.
- Application to thermal diffusion equations.





Any questions?

More details:

Master projects reports:

ronan-dupont.github.io/publication/2021-master-reports

Table of Contents

Modelling beaches morphodynamic by Hadamard sensitivity analysis

A subject aimed at developing a new, highly multidisciplinary morphodynamic model :

- From mathematics (optimization, calculation of partial derivatives, ...)
- From numerical mathematics (numerical schemes, finite element methods, finite volumes, numerical optimization: gradient descent, genetic methods)
- From marine science (linear theory, wave models, ...)
- From computer development (code in Python, Fortran, use of solvers such as PETSc Solver)

Publications:

Ronan Dupont, Frédéric Bouchette, and Bijan Mohammadi. (2024). "Beaches morphodynamic modeling based on Hadamard sensitivity analysis." In: *Ocean Modelling*, doi.org/10.1016/j.ocemod.2024.102370.

Ronan Dupont, Megan Cook, Frédéric Bouchette, Bijan Mohammadi, and Samuel Meulé. (2023). "Sandy beach dynamics by constrained wave energy minimization." In: *Ocean Modelling*, doi.org/10.1016/j.ocemod.2023.102197.

Modelling beaches morphodynamic by Hadamard sensitivity analysis

Motivations:

- Understand and anticipate coastal phenomena such as erosion.
- Contribute to the advancement of coastal numerical modeling.
- Reproduce a phenomenon that is rarely reproduced in morphodynamic models (creation of a sediment bar).
- Explore a new way to model sand dynamics with a limited number of hyperparameters.
- Develop a fast-executing tool for designing coastal defense structures.

Main model assumption

Nature adapts the seabed ψ so that wave energy is minimized E_H, ie:

$$\min_{\psi} \mathcal{J}(\psi) \quad \text{with} \quad \mathcal{J}(\psi) = \int_{\Omega} E_H(\psi),$$

Workflow based on the Hadamard shape derivative

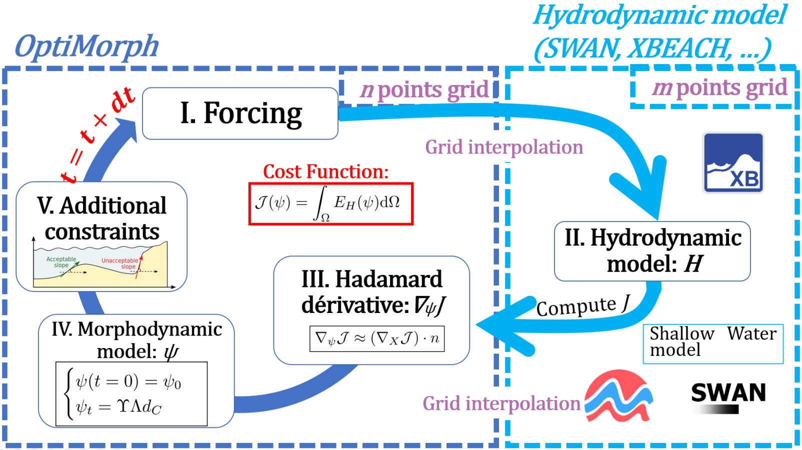


Figure: OptiMorph workflow based on Hadamard shape derivative.

Morphodynamic model

Problem reminder:

We aim to solve $\min_{\psi} \mathcal{J}(\psi)$ under physical constraints.

Without physical constraints:

$$\begin{cases} \psi_t = \Upsilon d \\ \psi(t=0) = \psi_0 \end{cases}$$

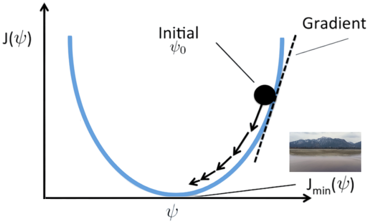


Figure: *Gradient descent at $t = 0$ s.*

where:

- $\psi(t)$: seabed profile at time t,
- Υ: sediment mobility,
- d: descent direction. Here, $d = -\nabla_{\psi}\mathcal{J}$.

Morphodynamic model

Problem reminder:

We aim to solve $\min_{\psi} \mathcal{J}(\psi)$ under physical constraints.

Without physical constraints:

$$\begin{cases} \psi_t = \Upsilon d \\ \psi(t=0) = \psi_0 \end{cases}$$

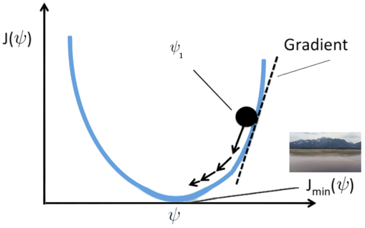


Figure: Gradient descent at $t = \Delta t$ s.

where:

- $\psi(t)$: seabed profile at time t,
- Υ: sediment mobility,
- d: descent direction. Here, $d = -\nabla_{\psi}\mathcal{J}$.

Morphodynamic model

Problem reminder:

We aim to solve $\min_{\psi} \mathcal{J}(\psi)$ under physical constraints.

With physical constraints:

$$\begin{cases} \psi_t = \Upsilon \cancel{d}\ d_C \\ \psi(t=0) = \psi_0 \end{cases}$$

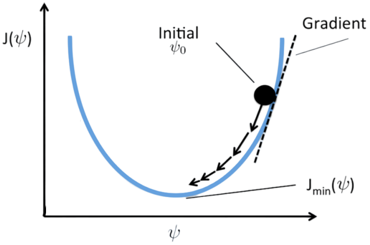


Figure: Gradient descent with constraints.

where:

- $\psi(t)$: seabed profile at time t,
- Υ: sediment mobility,
- d_C: descent direction including constraints.

Morphodynamic model

Problem reminder:

We aim to solve $\min_{\psi} \mathcal{J}(\psi)$ under physical constraints.

With physical constraints:

$$\begin{cases} \psi_t = \Upsilon \Lambda d_C \\ \psi(t=0) = \psi_0 \end{cases}$$

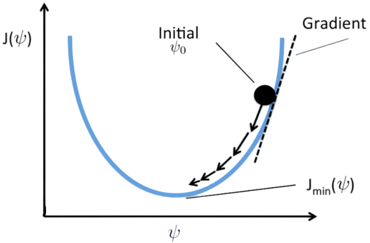


Figure: Gradient descent.

where:

- $\psi(t)$: seabed profile at time t,
- Υ: sediment mobility,
- d_C: descent direction including constraints,
- $\Lambda(x)$: wave-induced excitation of the seabed.

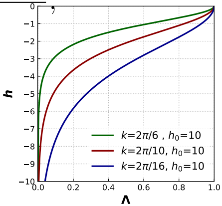
Adding physical constraints

- Sand excitation. From (Soulsby, 1987):

$$\varphi : \Omega \times [0, h_0] \longrightarrow \mathbb{R}^+$$
$$(x, z) \longmapsto \frac{\cosh(k(x)(h(x) - (h_0 - z)))}{\cosh(k(x)h(x))},$$

and at $z = \psi$:

$$\Lambda(x) = \varphi(x, \psi(x)) = \frac{1}{\cosh(k(x)h(x))}$$



- Maximum slope:

$$\left| \frac{\partial \psi}{\partial x} \right| \leq M_{\text{slope}}$$

- Sandstock conservation (in closed domain only):

$$\int_\Omega \psi(t, x) \mathrm{d}x = \int_\Omega \psi_0(x) \mathrm{d}x \quad \forall t \in [0, T_f]$$

Projection of the sediment conservation constraint

We aim to compute the descent direction d_C without violating the sediment conservation constraint. We define a residue $C_{\text{sand}}(t)$ which we want to be zero $\forall t \in [0, T_f]$.

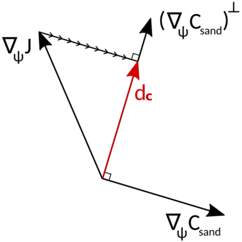


Figure: Projection. Cook (2021).

So we have:

$$d_C = \nabla_\psi \mathcal{J} - \left\langle \nabla_\psi \mathcal{J}, \frac{\nabla_\psi C_{\text{sand}}}{\|\nabla_\psi C_{\text{sand}}\|} \right\rangle \frac{\nabla_\psi C_{\text{sand}}}{\|\nabla_\psi C_{\text{sand}}\|}.$$

How to compute $\nabla_\psi \mathcal{J}$? Different strategies

Reminder of the governing equation:

$$\begin{cases} \psi_t = \Upsilon \wedge d_C \\ \psi(t=0) = \psi_0 \end{cases} \qquad \begin{cases} d_C = -\nabla_\psi \mathcal{J} + \text{constraints}, \\ \mathcal{J}(\psi) = \int_\Omega E_H(\psi) \end{cases}$$

Analytical Calculation

✔ Exact solution.
✔ Fast.

✘ Feasible only on simple models.

Cook (2021)

Finite Differences

✔ Easy to compute.

✘ Requires $N+1$ evaluations (each time step).
✘ Very long computation time.

Automatic Differentiation

Inria TAPENADE

✔ Robust.

✘ Heavy pre-processing.
✘ Depends on C / Fortran 90.

Mohammadi & Bouchette (2014)

Hadamard derivative of $\nabla_\psi \mathcal{J}$

We consider:

$$\nabla_\psi \mathcal{J} = \lim_{\varepsilon \to 0} \frac{\mathcal{J}(\psi + \varepsilon n) - \mathcal{J}(\psi)}{\varepsilon}$$

with n: normal vector to the shape.

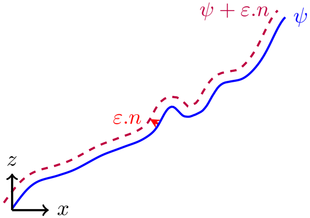


Figure: Illustration of Hadamard derivative.

At first order:

$$\nabla_\psi \mathcal{J} \approx \lim_{\varepsilon \to 0} \frac{\cancel{\mathcal{J}(\psi)} + \varepsilon \nabla_X \mathcal{J}.n - \cancel{\mathcal{J}(\psi)}}{\varepsilon}, \quad \text{with } X = (x\,,\,z)^\intercal\,.$$

This gives:

$$\boxed{\nabla_\psi \mathcal{J} \approx (\nabla_X \mathcal{J}) \cdot n}$$

Numerical validation on an analytical case

We consider:

$$\psi = \{(x, y) \in \mathbb{R}^2 \mid y = ax + b\}$$

and $\mathcal{J} = \cos(\psi)$, with $\boxed{\nabla_\psi \mathcal{J} = -\sin(\psi)\sqrt{a^2+1}}$.

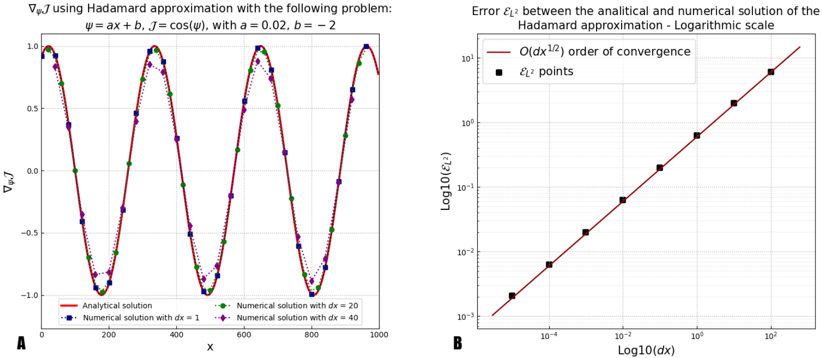


Figure: ***A)*** *Analytical and approximate solution using the derivative...*

Numerical verification on an application case with perturbations

With $\boxed{\nabla_\psi \mathcal{J} \approx \nabla_X \mathcal{J} \cdot n}$

Parameters:

- Shoaling model with:
  - $H_0 = 2$ m,
  - $T_0 = 10$ s,
  - $h_0 = 10$ m.
- Linear bottom with perturbations.

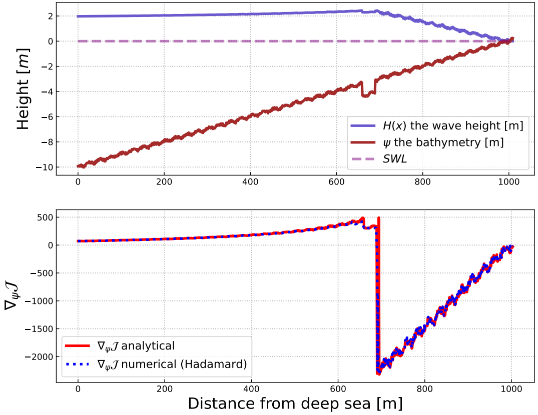


Figure: Computation of $\nabla_\psi \mathcal{J}$ using the Hadamard approach (blue) and analytical calculation (red). Linear bottom with perturbations.

Adding lateral transport

- New governing equation: $\boxed{\psi_t + V\nabla_s\psi = \Upsilon \Lambda d_C}$
- A possible velocity: $\boxed{V \sim U_b \left(\frac{H}{H_{max}}\right)^p}$ with $U_b = \frac{\pi H}{T_0 \sinh(kh)}$

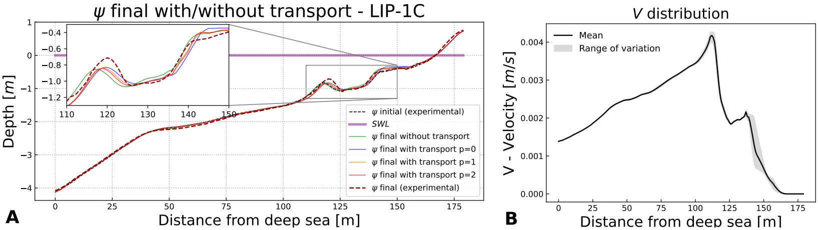


Figure: ***A)*** *OptiMorph morphodynamic results with lateral transport for $p = 0, 1, 2$, and XBeach wave model, for the LIP 1C channel experiment.* ***B)*** *Velocity distribution for $p = 1$.*

- But $V \geq 0$ for this choice $\Rightarrow$ no offshore sediment bar displacement (as in LIP-1B).

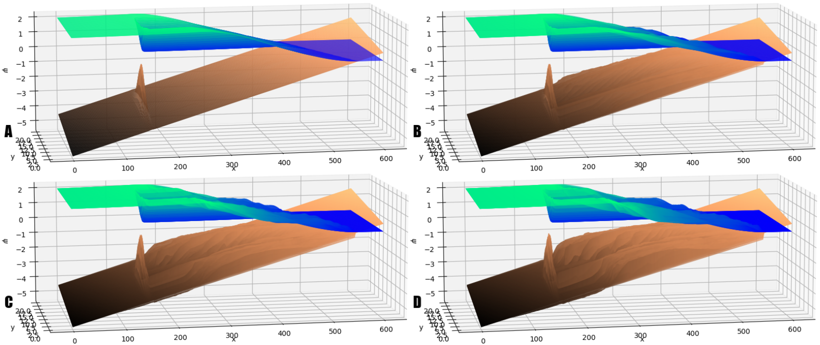
Towards a better definition of velocity?

- A new system of governing equations?

$$\begin{cases} \psi_t + V\nabla_s\psi = \Upsilon \Lambda d_C & \text{(a)} \\ V_t = -\rho \nabla_V \mathcal{J} & \text{(b)} \end{cases}$$

- Equation (b) represents a velocity that minimizes $\mathcal{J}$.
- How to compute $\nabla_V \mathcal{J}$?
- Can lateral morphodynamic transport minimize $\mathcal{J}$?

2D Morphodynamic results



- Observation of a sewage pit behind the geotextile tube.
- Seabed modification $\Rightarrow$ appropriate morphodynamic response.

Any questions?

More details:

- PhD manuscript:
  `ronan-dupont.github.io/publication/2024_manuscript`
- PhD defense (oral support):
  `ronan-dupont.github.io/talks/phd_defense`

Table of Contents

An arbitrary-order Virtual Element Method for the Helmholtz equation applied to wave field calculation in port

A subject to get me closer to numerical mathematics while retaining coastal skills.

- A Resolution of the Helmholtz Equation Using the Virtual Element Formalism.
- Any polyhedral mesh can be used, and a high-order calculation can be performed.
- A guide to Implementing the Virtual Element Method with a **Robin Boundary Condition**.
- A Concrete Application for Calculating Wave Eigenmodes of the Port of Cherbourg (France).

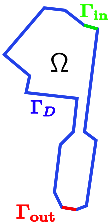
Publication:

Ronan Dupont. (2025). "An Arbitrary-Order Virtual Element Method for the Helmholtz Equation Applied to Wave Field Calculation in Port." *Results in Applied Mathematics*, doi.org/10.1016/j.rinam.2025.100598.

Problem configuration

A reference to my home town: Cherbourg in France.

Port location



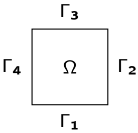
Port boundary

The Helmholtz or Mild-Slope equation:

$$\begin{cases} \nabla(C_p C_g \nabla a) + k^2 C_p C_g \, a = 0, & \text{in } \Omega \,, \\ \dfrac{\partial a}{\partial n} = 0, & \text{in } \Gamma_{in} \\ \dfrac{\partial a}{\partial n} + ik \, a = 0, & \text{in } \Gamma_{ou} \\ a = \gamma \, a_i & \text{in } \Gamma_D \end{cases}$$

Analytical solution

We consider,



$$\begin{cases} \Delta u + k^2\, u = f(x,y) & , \quad \text{in } \Omega\,, \\ u = u_{\text{exact}} & , \quad \text{on } \Gamma_2 \cup \Gamma_3 \cup \Gamma_4 \\ \dfrac{\partial u}{\partial n} + i\,k\,u = g(x,y) & , \quad \text{on } \Gamma_1, \end{cases}$$

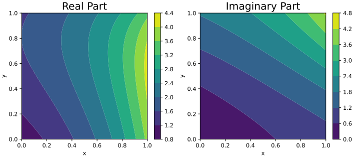


Figure: *Real and Imaginary part of* $u_{\mathbf{exact}}$.

with:

$$\begin{cases} u_{\text{exact}}(x,y) = (x+y)\cdot(1+i) + \exp(x^2 + i\,y^2), \\ f(x,y) = -\left((2x)^2 + (2\,i\,y)^2 + 2(1+i)\right)\cdot\exp(x^2 + i\,y^2) + k^2\cdot u_{\text{exact}}(x,y), \\ g(x,y) = (1+i) + (2\,i\,y)\cdot\exp(x^2 + i\,y^2) + i\,k\cdot u_{\text{exact}}(x,y). \end{cases}$$

Validation and Convergence

- Manufactured solution method: exact solution + right-hand side f.
- Implementation tested with $k = 1, 2, 3, 4, 5$ using quadrilateral and polygonal meshes.
- Observed convergence rate:
  - $\|u - u_h\|_{L^2} \sim \mathcal{O}(h^{k+1})$

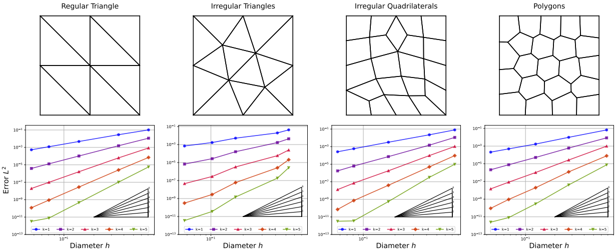


Figure: Convergence curves with different orders k and different types of elements.

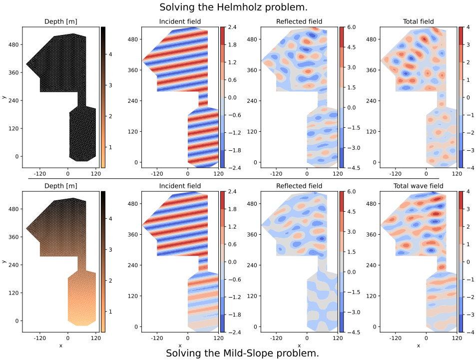
Slope sensitivity, Helmholtz vs Mild-Slope

Problem conditions:

- $a_{\max} = 2$ m,
- $T_0 = 8$ s,
- $\theta = 280\ °$.

Points of interest:

- Eigenmode position.



Any questions?

More details:

- Ronan Dupont. (2025). "An Arbitrary-Order Virtual Element Method for the Helmholtz Equation Applied to Wave Field Calculation in Port." *Results in Applied Mathematics*, doi.org/10.1016/j.rinam.2025.100598.
- Presentation at the NuMerics2024 conference where I was invited (oral support): `ronan-dupont.github.io/talks/NuMerics2024`